Future Value of an Ordinary Annuity and Sinking Funds

An annuity is a payment of money over equal intervals of time. Ordinary annuities and annuities due are terms used to describe the future values of annuities. The future value of an annuity is the total value of the contract at the end of the period of time if a fixed amount is deposited at specific intervals.

A sinking fund is initiated if a company anticipates buying an expensive piece of equipment at some time in the future.

Objectives:

- Determine the future value of an annuity or a sinking fund if a fixed amount is deposited at specific intervals.
- Determine the fixed amount of the deposit required at specific intervals to attain a desired future value of an annuity or a sinking fund.

Example 1:

A chemist deposits $300 in a savings account that pays 4% interest compounded annually and adds $300 at the end of each year for 5 years. How much money does she have after 5 years?

1. Press ⎡apps⎤ and select **Finance**. Press ⎡enter⎤ to select **TVM Solver** from the CALC menu

   Note: The mode DECIMAL SETTING was changed to **FIX2** to round computations to two decimal places.

   N is the number of payment periods, **I%** is the annual interest rate, **PV** is the Present Value, **PMT** is the Payment Amount, **FV** is the Future Value, **P/Y** is the number of payment periods per year, and **C/Y** is the number of compounding periods per year. The last row, **PMT:**, indicates whether payments are made at the end or beginning of each payment period.

2. Enter N = 5, I% = 4, PV = -300, PMT = -300 (because the flow of the money is from the saver to the bank), P/Y = 1, and C/Y = 1.

3. Place the cursor on the FV. Press ⎡alpha⎤ ⎡solve⎤. Notice that a square beside FV indicates that the problem has been solved.

The chemist has $1,989.89 after 5 years.
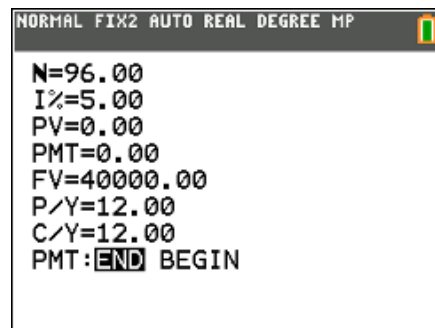
Example 2:

Grandparents of a fourth grader decided to start a college fund so that in 8 years their grandchild will have $40,000 saved toward college tuition. What monthly payments must they make if they find a bank paying 5% interest?

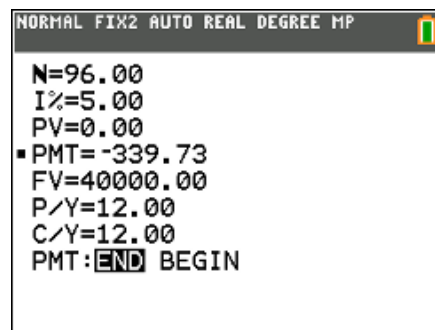Follow Steps 1-3 in **Example 1** above, using the values for this example in place of those above.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=96.00
 I%=5.00
 PV=0.00
 PMT=0.00
 FV=40000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Note that PMT is negative 339.73 because this is the amount the grandparents must pay out monthly.
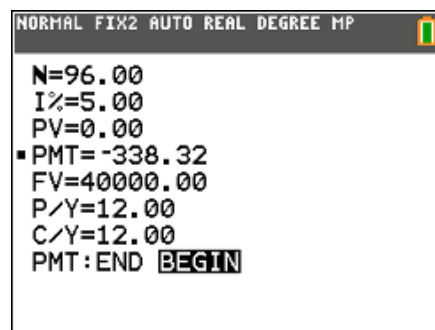
```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=96.00
 I%=5.00
 PV=0.00
■PMT=-339.73
 FV=40000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Change PMT: to **BEGIN**, and calculate the payment again.

Note that if the payments are made at the beginning of each period, the payment would be $338.32 and the $40,000 would be available one month after the last payment.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=96.00
 I%=5.00
 PV=0.00
■PMT=-338.32
 FV=40000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Example 3:

A 55 year-old man would like to have $100,000 in his account when he retires in 10 years. What monthly payments should he make to an account that pays 6% monthly?

Method 1: Using the **TVM Solver**

1. Press $\boxed{\text{apps}}$ and select **Finance**. Press $\boxed{\text{enter}}$ to select **TVM Solver**.

2. Enter N = 120, I% = 6, PV = 0, FV = 100000, P/Y = 12, and C/Y = 12.

3. Move the cursor to PMT, and press $\boxed{\text{alpha}}$ $\boxed{\text{solve}}$.

With the payments made at the end of each month, the monthly payments will be $610.21.

```
NORMAL FIX2 AUTO REAL DEGREE MP

N=120.00
I%=6.00
PV=0.00
▪PMT=-610.21
FV=100000.00
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
```
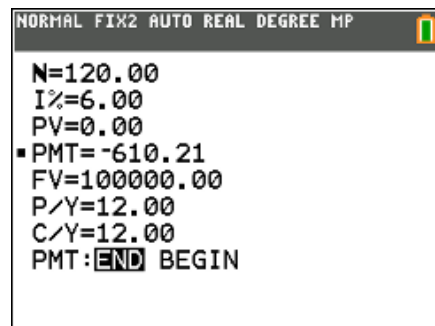
Method 2: Using the **tvm_Pmt** function

1. Press $\boxed{\text{2nd}}$ $\boxed{\text{quit}}$ to return to the home screen.

2. Press $\boxed{\text{apps}}$, select **Finance**, and choose **tvm_Pmt** from the CALC menu.

    This will paste the **tvm_Pmt** function on the home screen.

    The syntax for this function is **tvm_Pmt**(N, I%, PV, FV, P/Y, C/Y).

3. Enter $\boxed{(}$ 120 $\boxed{,}$ 6 $\boxed{,}$ 0 $\boxed{,}$ 100000 $\boxed{,}$ 12 $\boxed{,}$ 12 $\boxed{)}$ $\boxed{\text{enter}}$.
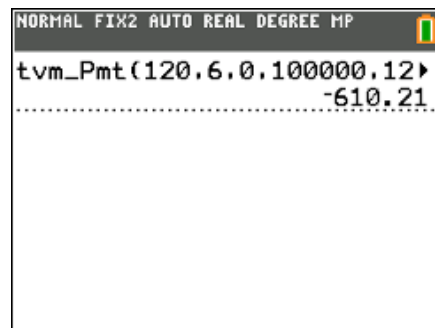
```
NORMAL FIX2 AUTO REAL DEGREE MP

tvm_Pmt(120,6,0,100000,12▸
                  -610.21
```

This matches the answer obtained from the **TVM Solver** with the payment at the END of the period.

Example 4:

A mathematics teacher deposits $1,000 in a savings and loan account at the end of each quarter for 10 years. How much money does he have at the end of 10 years if the savings and loan pays 4.5% interest compounded quarterly?
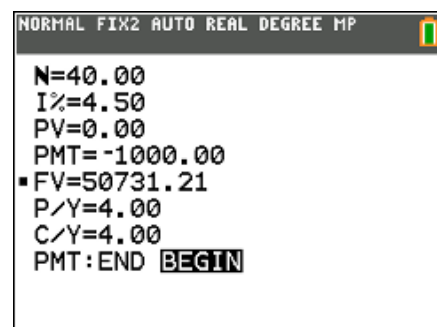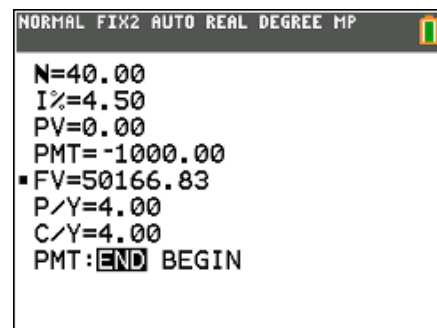
Method 1: Using the **TVM Solver**

1. Press [apps] and select **Finance**. Press [enter] to select **TVM Solver**.

2. Enter the values as in previous examples and press [alpha] [solve].

The teacher will have $50,166.83.

```
NORMAL FIX2 AUTO REAL DEGREE MP
N=40.00
I%=4.50
PV=0.00
PMT=-1000.00
▪FV=50166.83
P/Y=4.00
C/Y=4.00
PMT:END BEGIN
```

3. Change PTM to **BEGIN**, move the cursor back to FV and press [alpha] [solve].

If the teacher makes his deposits at the beginning of each compounding period, he will have $50,731.21, a difference of $564.38.

```
NORMAL FIX2 AUTO REAL DEGREE MP
N=40.00
I%=4.50
PV=0.00
PMT=-1000.00
▪FV=50731.21
P/Y=4.00
C/Y=4.00
PMT:END BEGIN
```

Method 2: Using the **tvm_FV** function

1. Change **PTM:** back to **END** in the **TVM Solver**.

2. Press [2nd] [quit] to return to the home screen.

3. Press [apps], select **Finance**, and choose **tvm_FV** from the CALC menu to paste the function on the home screen.

    The syntax for this function is **tvm_FV**(N, I%, PV, PMT, P/Y, C/Y).

4. Enter [( ] 40 [,] 4.5 [,] 0 [,] [(-)] 1000 [,] 4 [,] 4 [ )] [enter].

    The result is $50,166.83.

    This matches the answer obtained from the **TVM Solver** with the payment at the END of the period.

```
NORMAL FIX2 AUTO REAL DEGREE MP
tvm_FV(40,4.5,0,-1000,4,4)
                    50166.83
```
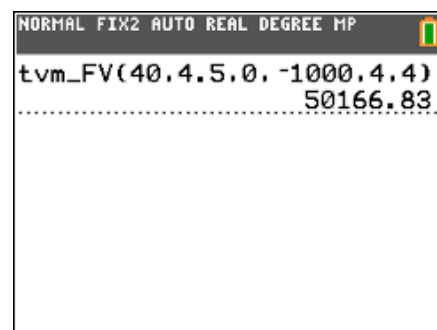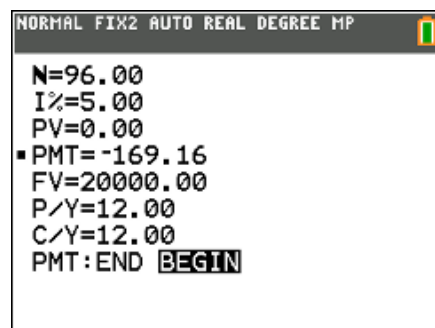
Example 5:

A family decides to make monthly deposits into a college education fund for a daughter so that she will have $20,000 at the end of 8 years. They locate a bond fund that pays 5% compounded monthly. How much must the family deposit each month?

Notice the difference of $0.71 each month if the payments are made at the beginning of the period versus at the end of the period.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=96.00
 I%=5.00
 PV=0.00
▪PMT=-169.87
 FV=20000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=96.00
 I%=5.00
 PV=0.00
▪PMT=-169.16
 FV=20000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```
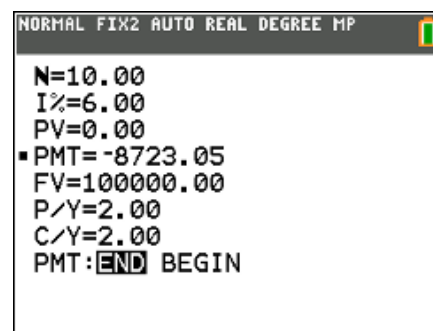
Sinking Funds

A sinking fund is initiated if a company anticipates buying an expensive piece of equipment at some time in the future. Periodic payments are made to an account so that the company can pay for the item rather than borrow the money to pay for it (amortization). Rather than paying interest on the money they would borrow, they accumulate interest on the money that they save each time period.

Example 1:

In five years, a company wants to buy a new computer system costing $100,000. They establish a sinking fund which pays 6% compounded semiannually. To accumulate $100,000 in five years, what is the payment every six months?

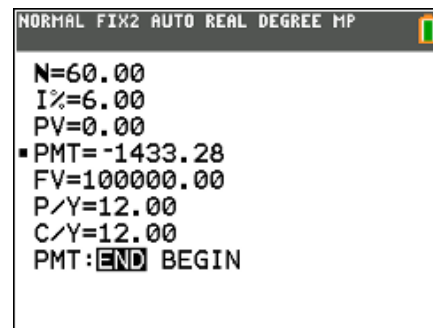The company must make ten semi-annual payments of $8,723.05.

Notice that PMT is negative because the money is flowing away from the company.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=10.00
 I%=6.00
 PV=0.00
■PMT=-8723.05
 FV=100000.00
 P/Y=2.00
 C/Y=2.00
 PMT:END BEGIN
```

Example 2:

A company wants $100,000 in a sinking fund after 5 years. The company is willing to make monthly payments because the investment will compound at 6% monthly. Find the amount of the payment that the company must make each month.

The company must make monthly payments of $1,433.28.

```
NORMAL FIX2 AUTO REAL DEGREE MP

 N=60.00
 I%=6.00
 PV=0.00
■PMT=-1433.28
 FV=100000.00
 P/Y=12.00
 C/Y=12.00
 PMT:END BEGIN
```

Example 3:

A manufacturer deposits $1,000 each month in an account which pays 5% interest compounded monthly. She anticipates doing this for the next 5 years. How much money will be in her sinking fund to buy an updated machine to cap bottles at the end of that time period?

Her sinking fund will accumulate $68.006.08.

```
NORMAL FIX2 AUTO REAL DEGREE MP

N=60.00
I%=5.00
PV=0.00
PMT=-1000.00
▪FV=68006.08
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
```